RESEARCH DEPARTMENT

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SUMMARY

The charge time-constant of a peak rectifier is commonly specified in terms of the rate of rise of voltage with time when a sinusoidal signal is suddenly applied. An expression for rise time is derived, particularly for the case in which the time of one cycle of a.c. is small compared with the charge time-constant of the rectified system; the time then taken for the rectified voltage to reach any given percentage of its steady-state value is shown graphically. In particular, an a.c. signal takes 2·5 times as long to reach 80% of its steady-state value as a d.c. step of the same amplitude.

1. INTRODUCTION

It is sometimes necessary to check experimentally the charge time-constant of a rectifier system designed to register the crest value of a signal wave form. The required information can be obtained by suddenly applying a d.c. voltage and determining the time required for the voltage across the reservoir capacitor to attain a specified fraction - for example 80% - of its steady-state value. It frequently happens, however, that the test signal, before reaching the rectifier system, has to pass through intermediate circuits whose pass band does not extend to zero frequency and which are therefore incapable of transmitting a sustained d.c. step. It is therefore common practice to substitute for the d.c. test signal a suddenly applied a.c. signal of such a frequency that the time of one cycle is small compared with the charge time-constant to be measured. Since, with the a.c. signal, the rectifier system is operative only during part of each cycle, the time taken to attain a specified fraction of the steady-state voltage is clearly longer than that required with a d.c. step, but the relationship between the two quantities has not, as far as is known, been calculated. In this report, a general expression, applicable to both the d.c. and a.c. cases, is derived for the rise of rectified voltage as a function of time. The internal resistance of the rectifier system has been regarded as constant in what follows, although in practice this resistance tends to increase without limit as the applied voltage tends to zero.

The system under consideration is equivalent to a *CR* circuit as in Fig. 1, to which a voltage is applied by means of another circuit (not shown) containing

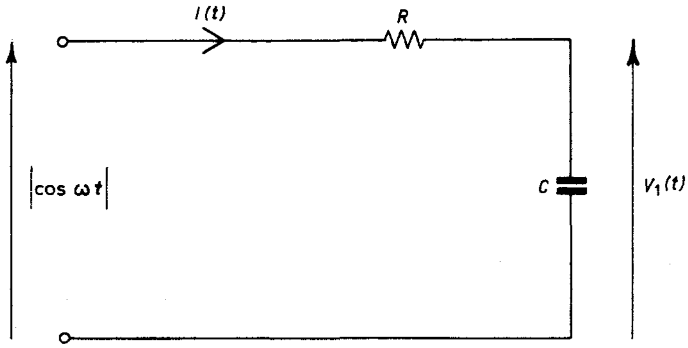

Fig. 1

Essentials of the circuit

rectifiers, in such a way that the input voltage is $\cos \omega t$ when this quantity is positive and $\cos(\omega t + \pi)$ when $\cos \omega t$ is negative. If the input voltage is less than the voltage present at the same time on C, no current flows and $V_1(t)$ remains constant, but if the input voltage is greater than that on C, a positive current $I(t)$ flows, and charges C so that $V_1(t)$ increases. We have to consider the change in $V_1(t)$ during a (positive) half cycle, in order to deduce the time taken by the voltage on the capacitor to reach any given percentage of its ultimate value, unity, at all frequencies.

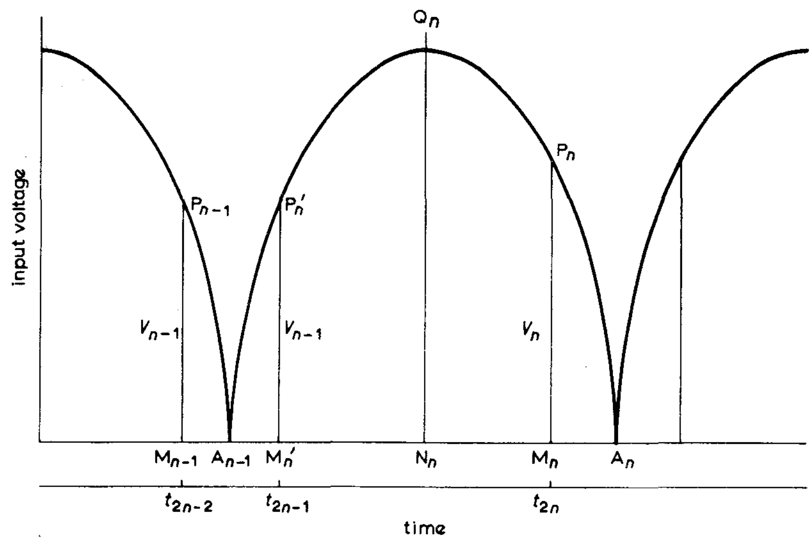

Fig. 2 - General behaviour of input and capacitor voltages

Suppose that nth half cycle is a positive one as indicated in Fig. 2. (There is no essential difference between positive and negative half cycles, but there are several places where the angle ωt in the argument which follows has to be replaced by $\omega t + \pi$). At a certain time, which we have called t_{2n-2} and represented by the point M_{n-1} in Fig. 2, the falling input voltage $\cos(\omega t + \pi)$ in the negative $(n\text{-}1)$th half cycle will have descended to equal the voltage V_{n-1} already on C. V_{n-1} is represented by the ordinate $P_{n-1}M_{n-1}$ in Fig. 2. Thereafter $I(t)$ becomes zero, and $V_1(t)$ remains constant, until the rising voltage $\cos \omega t$ of the nth half cycle reaches the value V_{n-1} at a time t_{2n-1} represented by M_n' in Fig. 2. Then $I(t)$ becomes positive, and the voltage on C increases, until, in the latter part of the nth half cycle, the input voltage falls to the increased voltage V_n which has been reached

by C as a result of the charging by $I(t)$ in the nth half cycle. In Fig. 2 this cessation of $I(t)$ is shown as occurring at a time t_{2n} (represented by the point M_n in Fig. 2) and the increased voltage V_n then on C is represented by the ordinate P_nM_n. We have to determine the change $V_n - V_{n-1}$ in $V_1(t)$ during the nth half cycle.

2. THEORETICAL ANALYSIS

At a time t between t_{2n-1} and t_{2n}, the situation in Fig. 1, which is a linear circuit, can be regarded as the superposition of (a) a constant input voltage V_{n-1} which balances the voltage V_{n-1} already on the capacitor, so that no current flows and the charge on C remains constant, (b) a voltage $\cos \omega t - V_{n-1}$ applied to the circuit of Fig. 1, regarded as 'dead', at time t_{2n-1}. In case (b), Ohm's Law applied to Fig. 1 gives

$$\cos \omega t - V_{n-1} = \left(R + \frac{1}{\mathrm{p}C}\right) I(t) \tag{1}$$

and

$$I(t) = \mathrm{p}C\, V_1(t) \tag{2}$$

In the total situation, $I(t)$ is the total current, but the total voltage on C is $V_1(t) + V_{n-1}$.

Eliminating $I(t)$ between Equations (1) and (2)

$$V_1(t) = \frac{1}{1 + \mathrm{p}CR} (\cos \omega t - V_{n-1}) \, . \tag{3}$$

so that, if $\alpha = 1/(CR)$,

$$V_1(t) = \frac{\alpha}{\mathrm{p} + \alpha} (\cos \omega t - V_{n-1}) \tag{4}$$

$$= \alpha e^{-\alpha t} \int_{t_{2n-1}}^{t} e^{\alpha \tau} \, [\cos \omega \tau - V_{n-1}] \, d\tau \tag{5}$$

$$= \alpha e^{-\alpha t} \left[e^{\alpha \tau} \left(\frac{\alpha \cos \omega \tau + \omega \sin \omega \tau}{\alpha^2 + \omega^2} - \frac{V_{n-1}}{\alpha} \right) \right]_{t_{2n-1}}^{t} \tag{6}$$

$$= \frac{\alpha}{\alpha^2 + \omega^2} \begin{bmatrix} \alpha \cos \omega t + \omega \sin \omega t \\ - e^{-\alpha(t - t_{2n-1})} \{\alpha \cos \omega t_{2n-1} + \omega \sin \omega t_{2n-1}\} \end{bmatrix}$$

$$- V_{n-1} \{1 - e^{-\alpha(t - t_{2n})}\} \tag{7}$$

Our objective is to determine the behaviour of $V_1(t)$ at high frequencies and compare this with the behaviour at zero frequency. Consider therefore first the situation at zero frequency. In this case $\omega = 0$, and there is only a single incomplete positive half cycle, which can be regarded as beginning at the time, say t_0, of maximum voltage input represented by N_n in Fig. 2, with a unit-step input. We must take n as unity, and the initial voltage V_{n-1} or V_0, on the capacitor, is zero. Equation (7) reduces to

$$V_1(t) = 1 - e^{-\alpha(t-t_0)} \tag{8}$$

where t_0 is the starting time. Hence the time T required for $V_1(t)$, which in this case is the total voltage on C, to reach the value x ($100x\%$ of the ultimate value 1) is given by

$$e^{-\alpha(T-t_0)} = 1-x$$

or

$$T-t_0 = \frac{-\log_e (1-x)}{\alpha} \tag{9}$$

In particular if $x = 0{\cdot}8$, $-\log_e (1-x) = 1{\cdot}60944$.

If ω/α is small or comparable with unity, Equation (7) will tell us the change in $V_1(t)$ in each successive half cycle, and it will be necessary to pay attention to the starting time (t_1 in the notation of Fig. 2, with $V_0 = 0$) which may be anywhere in that half cycle, and consider each half cycle separately. The rise of $V_1(t)$ with t will consist of a series of upward curves separated by horizontal straight portions, since there will be increasing intervals between successive half cycles during which no current $I(t)$ is flowing. But the case we are primarily concerned with is when $\omega \gg \alpha$, so that the contribution per half cycle to $V_1(t)$ is small, and we can take $V_{n-1} \approx V_n$, or more accurately, we can take $t_{2n} - t_{2n-2}$ as a complete half cycle π/ω, and $(V_n - V_{n-1})/(\pi/\omega)$ as dV_n/dt. As the contribution per half cycle to $V_1(t)$ is small, we can ignore the part of the first half cycle at which the time t_1 occurs. The voltage V_n on C at the end of half cycle n is then also $V_{n-1} + V_1(t_{2n})$ where $V_1(t)$ is given by Equation (7) with t put equal to t_{2n}. We thus obtain, for $\omega \gg \alpha$

$$\frac{dV_n}{dt} \approx \frac{V_n - V_{n-1}}{\pi/\omega}$$

$$\approx \frac{\alpha}{\pi} [\sin \omega t_{2n} - e^{-\alpha(t_{2n} - t_{2n-1})} \sin \omega t_{2n-1} - V_n \, \omega \, (t_{2n} - t_{2n-1})] \tag{10}$$

In Equation (10) we have ignored terms of order $1/\omega$ on the right hand side, including the difference between V_n and V_{n-1}. We can therefore regard $\exp \{-\alpha(t_{2n} - t_{2n-1})\}$ as unity, but we have to determine more accurately $\omega(t_{2n} - t_{2n-1})$ which is of order zero in ω.

Now at time t_{2n}, $\cos \omega t_n = V_n$ and at time t_{2n-1}, $\cos \omega t_n = V_{n-1} \approx V_n$ also, and t_{2n} and t_{2n-1} are in the same half cycle, so that $\omega(t_{2n} - t_{2n-1})$ can be taken as twice the acute angle $(\pi/2 - \theta)$ whose cosine is V_n. If now we write Equation (10) in terms of θ, noting that

$$\begin{aligned} V_n &= \sin\theta; & \sin \omega t_{2n} &= (1 - V_n^2)^{1/2} \\ & & &= \cos\theta \\ \frac{dV_n}{dt} &= \cos\theta \frac{d\theta}{dt}; & \sin \omega t_{2n-1} &= -(1 - V_n^2)^{1/2} \\ & & &= -\cos\theta \end{aligned} \tag{11}$$

Equation (10) becomes

$$\cos\theta \frac{d\theta}{dt} = \frac{2\alpha}{\pi} \cos\theta - \frac{\alpha \sin\theta}{\pi} [\pi - 2\theta] \tag{12}$$

Equation (12) is independent of ω, and can be reduced to an explicit integration. If, as before, T is the time of reading $100x\%$ of the ultimate value and t_0 is the starting time, Equation (12) becomes

$$\alpha(T - t_0) = \int_0^{\sin^{-1}x} \frac{d\theta}{(2/\pi) - [1-(2\theta/\pi)] \tan\theta} \tag{13}$$

We now have to compare the rise time $R_1(x)$ required to reach $100x\%$ of the ultimate value (unity) in response to a d.c. unit step, and the rise time $R_2(x)$ to reach the same percentage of the same ultimate value in response to an a.c. input. $R_1(x)$ is $(T-t_0)$ given by Equation (9) and $R_2(x)$ is $(T-t_0)$ given by Equation (13). These quantities are tabulated in Table 1 and plotted in Fig. 3. In both cases the natural units of time (which are used in Fig. 3) are inversely proportional to α, and therefore directly proportional to CR, the time constant of the circuit of Fig. 1. We are not from the practical point of view concerned with values of x above 0·95, though for the sake of completeness the behaviour of $R_1(x)$ and $R_2(x)$ as $x \to 1$ is briefly considered below. The labour of calculation of $R_2(x)$ is simplified if $\theta = \sin^{-1}x$ rather than x itself is a round number, so Simpson's Rule was applied with ordinates of θ at 0(0·05)0·4(0·02)0·88(0·01)1·26. $R_1(x)$ is easily calculated direct from Equation (9) for the values of x for which it is convenient to calculate $R_2(x)$. In Table 1 the ratio $R_2(x)/R_1(x)$ is also given. Since x is commonly taken as 0·8 to define the rise time, $\alpha R_2(0{\cdot}8)$ was also calculated and found to be 4·0199, while $\alpha R_1(0{\cdot}8)$ is 1·6094; the ratio $R_2(0{\cdot}8)/R_1(0{\cdot}8)$ is thus 2·498. $R_1(x)$ from (9) tends logarithmically to infinity, but it can be shown that if x_1 and x_2 are both near 1

$$R_2(x_2) - R_2(x_1) \approx \frac{3\pi}{2\sqrt{2}} \{(1-x_2)^{-1/2} - (1-x_1)^{-1/2}\}$$

so that $R_2(x)$ goes to infinity as $(1-x)^{-1/2}$ when $x \to 1$ and therefore the ratio $R_2(x)/R_1(x)$ also tends to infinity as $x \to 1$.

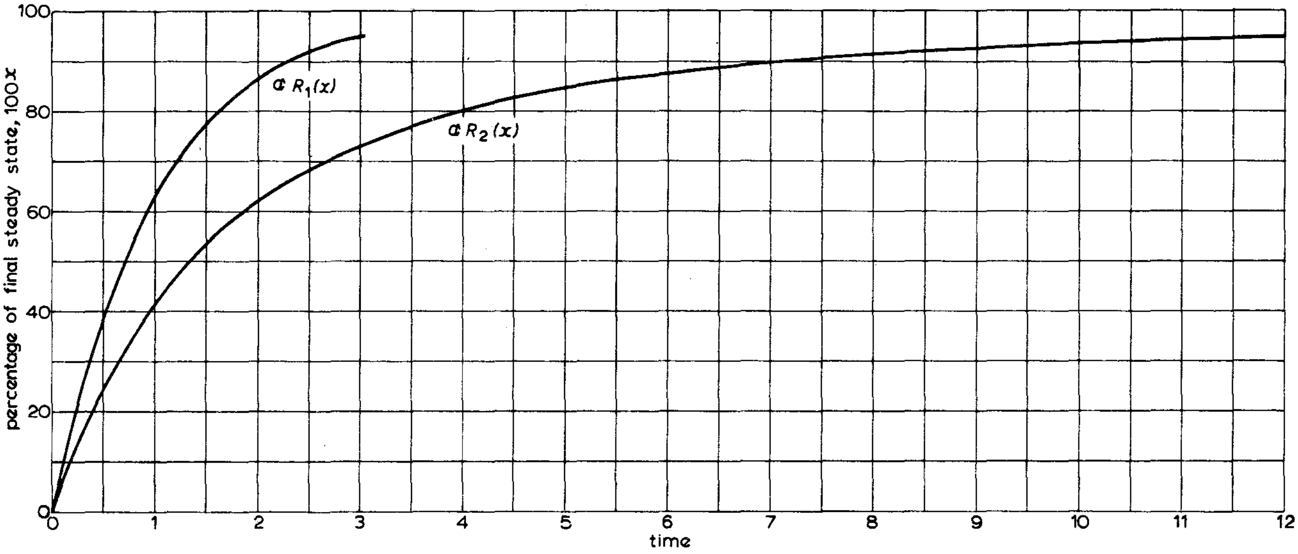

Fig. 3 - Variation of the charge on the capacitance C in Fig. 1 with time for unit-step input and for unit-amplitude high-frequency input respectively

3. CONCLUSIONS

If $R_1(x)$ is the time taken for the capacitor in the circuit of Fig. 1 to acquire a voltage which is $100x\%$ of the theoretical ultimate response to a d.c. unit-step input, and $R_2(x)$ is the corresponding time when the input is a rectified sine-wave of unit amplitude and of sufficiently high frequency, then $R_2(x)/R_1(x)$ is $\pi/2$ for sufficiently small x, increases steadily to about $2{\cdot}5$ when $x = 0{\cdot}8$, and increases without limit as x tends to 1. This behaviour of $R_2(x)/R_1(x)$ is independent of frequency provided that one half cycle of the rectified sine-wave takes a time small compared with the time-constant associated with Fig. 1.

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TABLE 1

$R_1(x)$, $R_2(x)$ and $R_2(x)/R_1(x)$ for various values of x

				(sin 0·3)	(sin 0·4)	(sin 0·44)				
x*	0	0·09983	0·19867	0·29552	0·38942	0·42594	0·46178	0·49688	0·53119	0·56464
$\alpha R_1(x)$	0	0·10517	0·22148	0·35029	0·49334	0·55502	0·61949	0·68693	0·75756	0·83158
$\alpha R_2(x)$	0	0·17024	0·37056	0·60796	0·89184	1·02111	1·16080	1·31225	1·47675	1·65591
$R_2(x)/R_1(x)$	Tends to $\pi/2$	1·6187	1·6731	1·7356	1·8078	1·8398	1·8738	1·9103	1·9494	1·9913

x*	0·59720	0·62879	0·65938	0·68892	0·71736	0·74464	0·77074	0·79560	0·81919	0·84147
$\alpha R_1(x)$	0·90931	0·99100	1·07697	1·16769	1·26360	1·36508	1·47280	1·58768	1·71032	1·84182
$\alpha R_2(x)$	1·85161	2·06607	2·30194	2·56237	2·85118	3·17303	3·53355	3·94010	4·40158	4·92955
$R_2(x)/R_1(x)$	2·0363	2·0848	2·1374	2·1944	2·2564	2·3244	2·3992	2·4817	2·5735	2·6765

					(sin 1·20)	(sin 1·24)	(sin 1·26)
x*	0·86240	0·88196	0·90010	0·91680	0·93204	0·94578	0·95209
$\alpha R_1(x)$	1·98340	2·13674	2·30358	2·48650	2·68884	2·91472	3·0384
$\alpha R_2(x)$	5·53894	6·24975	7·08905	8·09446	9·31946	10·84021	11·75078
$R_2(x)/R_1(x)$	2·7926	2·9249	3·0774	3·2554	3·4660	3·7191	3·8674

* These values of x are chosen so that $\sin^{-1}x$ shall be a round number of radians, which simplifies the numerical calculation required. The values of $\sin^{-1}x$ are: 0(0·1)0·4(0·04)1·24 and 1·26.

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